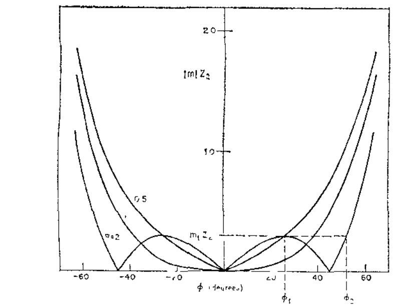

Fig. 2—Variation of m with ϕ for various values of R.

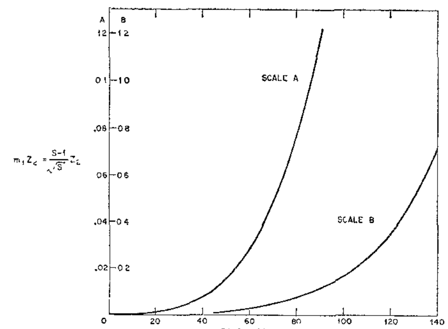

Fig. 3—Variation of m_1Z_2 with bandwidth.

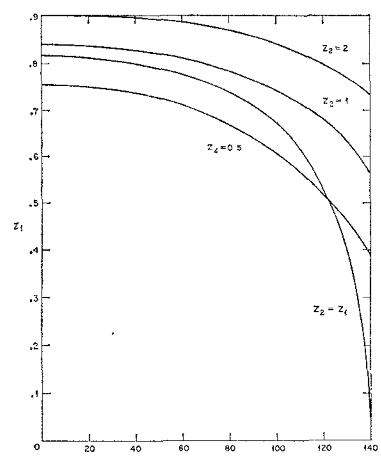

Fig. 4—Z_1 as a function of bandwidth for various stub impedances.

Although the above analysis assumed a coaxial stub it is obvious that the analysis can be applied to other TEM transmission lines and to waveguides. In particular it may be applied to the problem of making an extremely broad-band T junction for a branched duplexer.

An Extremely Broad-Band Rotary Joint

Electrically a choke type rotary joint consists of an open-circuited quarter-wavelength stub in series with a transmission line. A comparison with the broad-band stub of the previous section, which is a short-circuited quarter-wavelength stub in parallel with the line, suggests that an analysis of the choke type rotary joint on an admittance

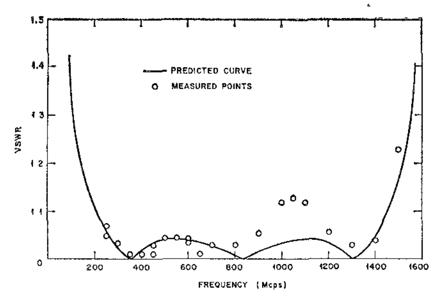

Fig. 5—Predicted and measured VSWR of broad-band rotary joint.

basis may give a result similar to the above analysis. This indeed is the case. If Z_1 and Z_2 in (1) through (7) are replaced by Y_1 and Y_2, respectively, these equations will give the response of a choke type rotary joint and at the same time will point out a method of broadbanding such a rotary joint. Broadbanding may be achieved by reducing the characteristic admittance of the transmission line by the proper amount for a quarter wavelength on either side of the chokes. Physically, this may be accomplished by decreasing the radius of the inner conductor or increasing the radius of the outer conductor of a coaxial line rotary joint.

A broad-band rotary joint using the above theory has been built and tested in three and one-eighth inch coaxial line. The predicted and measured results are shown in Fig. 5. The measured results agree quite well with the theory except in the region near 1100 mc. This can be explained by the lack of the theory in accounting for the capacitive discontinuity at the end of the series choke in the inner conductor and the effect of the short-circuited high impedance quarter-wave section at the end of the series choke in the outer conductor.

The sum of the characteristic impedances of the inner and outer chokes was 3.3 ohms and the main line had an impedance of 50 ohms. The rotary joint was designed to have a VSWR less than 1.04 over a 135 per cent bandwidth. For the same VSWR with no compensation the bandwidth would have been 70 per cent.

In conclusion the above analysis may be used to broadband any quarter-wavelength choke or stub type discontinuity and accurately predict its performance.

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Comments on Ozaki's Comments*

Ozaki's[1] comments have drawn my attention to the fact that there is a significant difference between "The Synthesis Theorem" given in my paper[2] and the restatement of it that is given and proven in the Appendix. If we define l to be the degree, the maximum of the degrees of the numerator and the denominator, of the rational function $Z(p)$, and r to be the number of line sections in the impedance transformer, then in the first statement of the theorem, l is unspecified and $n=r$ while in the second statement $l=n$ and r is unspecified. Now the second theorem is correct, even in view of Ozaki's comments, and accordingly is adequate for a proof of the physical realizability of the allowed insertion loss functions. The first theorem, however, is incorrectly stated as Ozaki's example has shown.

Ozaki's third condition, "Assuming that the numerator and denominator of $Z(p)$ in (1) are prime to each other, the degrees of both the numerator and denominator must be equal to n," correctly requires that $l=n$ and adds the restriction that the numerator and denominator of $Z(p)$ contain no common factors.

The requirement that the degree of the numerator of $Z(p)$ equal the degree of the denominator is a salient feature of the theory. My failure to define $l=n$, which has this consequence when taken with condition 2, in the first statement of the theorem, was simply an oversight. I permitted the removal of common factors[3] in the second statement of the theorem by not specifying r, since it is readily shown that the removal of a common factor from the numerator and denominator of a $Z(p)$, satisfying condition 2, results in a $Z'(p)$ which again satisfies this condition

Ozaki's third condition permits the proof of a sharper theorem than my application required, namely one in which $l=n=r$. His condition, however, is unnecessarily restrictive since the relative primeness of the numerator and denominator is not a necessary condition for the truth of this class of theorem. For example, if the terminating resistance, R, is preceded by a section of line of characteristic impedance, R, then the numerator and denominator of $Z(p)$ contain the common factor, $p+1$. In fact it is readily demonstrated that the only common factors permitted by condition 2 are products of $p+1$ and $p-1$. The first can be realized while the occurrence of the latter would result in the indeterminacy of $Z(1)$.

A more general theorem of this type can be stated:

The necessary and sufficient conditions that a rational function of p, determinant for $p=1$, with real coefficients, of degree at most n in numerator or denominator written in the form

$$Z(p) = \frac{m_1(p) + n_1(p)}{m_2(p) + n_2(p)}$$

with m_1 and m_2 odd or even and n_1 and n_2 even or odd, be the input impedance of a cascade of n equal-length transmission line sections terminated in a resistance are:

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[1] H. Ozaki, "On Riblet's theorem," IRE Trans. on Microwave Theory and Techniques, vol. MTT-6, pp. 331–332; July, 1958.

[2] H. J. Riblet, "General synthesis of quarter-wave impedance transformers," IRE Trans. on Microwave Theory and Techniques, vol. MTT-5, pp. 36–43; January, 1957.

[3] For example, the well-known result that a positive real function of p is a quotient of two Hurwitz polynomials is true in general only if the removal of common factors from numerator and denominator is permitted.

1) $Z(p)$ must be a positive real function of p;
2) $m_1(p)m_2(p)-n_1(p)n_2(p)=C(p^2-1)^n$.

Condition 2 implies that both numerator and denominator are of degree n and it is readily argued that an impedance function formed by terminating a section of transmission line in an indeterminant impedance function will remain indeterminant. Furthermore if $Z(p)$ is normalized so that the coefficient of p^n in its denominator is unity then C equals the terminating resistance.

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Vector Formulations for the Field Equations in Anisotropic Waveguides*

In the following we will exhibit vector formulations for the equations determining the different components of the electromagnetic field in a source-free uniform waveguide. All results will be stated without proof. The derivations are given elsewhere.[1] The vector formulations given below are applicable to uniform waveguides containing anisotropic media restricted only by the requirement that the permittivity ($\boldsymbol{\varepsilon}$) and permeability ($\boldsymbol{\mu}$) dyadics be independent of the axial coordinate z. For uniform waveguides (with the indicated restriction on $\boldsymbol{\mu}$ and $\boldsymbol{\varepsilon}$) we consider solutions to the Maxwell equations which display characteristic time and z dependence of the form $\exp i(\kappa z-\omega t)$. This assumption permits us to eliminate the z and t dependence from the Maxwell equations and rewrite these as:

$$\begin{bmatrix} \omega\boldsymbol{\epsilon} & -\boldsymbol{\nabla}_t\times\mathbf{1}-i\kappa\boldsymbol{z}_0\times\mathbf{1}_t \\ -\boldsymbol{\nabla}_t\times\mathbf{1}-i\kappa\boldsymbol{z}_0\times\mathbf{1}_t & \omega\boldsymbol{\mu}\end{bmatrix}\cdot\begin{bmatrix}\boldsymbol{E}\\ i\boldsymbol{H}\end{bmatrix}=0. \quad (1)$$

Here, as in all the matrix equations which follow, dot product multiplication is to be understood for the products of dyadics and vectors. In (1), $\boldsymbol{E}$ and $\boldsymbol{H}$ are, respectively, the steady-state electric and magnetic fields; $\boldsymbol{\nabla}_t$ is the transverse gradient operator; $\boldsymbol{z}_0$ is the unit vector in the axial direction; $\mathbf{1}$ is the unit dyadic; and $\mathbf{1}_t$ is the unit transverse dyadic:

$$\mathbf{1}_t=\mathbf{1}-\mathbf{1}_z=\mathbf{1}-\boldsymbol{z}_0\boldsymbol{z}_0. \quad (2)$$

It is well known that the transverse field components, $\boldsymbol{E}_t$ and $\boldsymbol{H}_t$, constitute the independent field components. To eliminate the dependent longitudinal components from (1) it is convenient to express, *e.g.*, the $\boldsymbol{\varepsilon}$ dyadic as

$$\boldsymbol{\varepsilon}\rightarrow\begin{bmatrix}\boldsymbol{\varepsilon}_t & \boldsymbol{\varepsilon}_{tz}\\ \boldsymbol{\varepsilon}_{zt} & \epsilon_z\end{bmatrix} \quad (3)$$

where $\boldsymbol{\varepsilon}_t$ is a transverse dyadic, $\boldsymbol{\varepsilon}_{tz}$ and $\boldsymbol{\varepsilon}_{zt}$ are vectors, and ϵ_z is a scalar; *i.e.*,

$$\boldsymbol{\varepsilon}=\boldsymbol{\varepsilon}_t+\epsilon_z\mathbf{1}_z+\boldsymbol{z}_0\boldsymbol{\varepsilon}_{zt}+\boldsymbol{\varepsilon}_{tz}\boldsymbol{z}_0. \quad (4)$$

A similar representation is chosen for the $\boldsymbol{\mu}$ dyadic. It can then be shown that the (independent) transverse field components satisfy the following pair of (coupled) second-order differential equations (transverse vector eigenvalue problem):

$$\begin{bmatrix}\left(\omega\boldsymbol{\varepsilon}_t-\frac{1}{\omega}\boldsymbol{\nabla}_t\times\boldsymbol{z}_0\frac{1}{\mu_z}\boldsymbol{z}_0\times\boldsymbol{\nabla}_t-\frac{\omega}{\epsilon_x}\boldsymbol{\varepsilon}_{tz}\boldsymbol{\varepsilon}_{zt}\right) & \left(\frac{\boldsymbol{\varepsilon}_{tz}}{\epsilon_z}\boldsymbol{z}_0\times\boldsymbol{\nabla}_t+\boldsymbol{\nabla}_t\times\boldsymbol{z}_0\frac{\boldsymbol{\mu}_{zt}}{\mu_z}-i\kappa\boldsymbol{z}_0\times\mathbf{1}_t\right)\\ \left(\frac{\boldsymbol{\mu}_{tz}}{\mu_z}\boldsymbol{z}_0\times\boldsymbol{\nabla}_t+\boldsymbol{\nabla}_t\times\boldsymbol{z}_0\frac{\boldsymbol{\varepsilon}_{zt}}{\epsilon_z}-i\kappa\boldsymbol{z}_0\times\mathbf{1}_t\right) & \left(\omega\boldsymbol{\mu}_t-\frac{1}{\omega}\boldsymbol{\nabla}_t\times\boldsymbol{z}_0\frac{1}{\epsilon_z}\boldsymbol{z}_0\times\boldsymbol{\nabla}_t-\frac{\omega}{\mu_z}\boldsymbol{\mu}_{tz}\boldsymbol{\mu}_{zt}\right)\end{bmatrix}\begin{bmatrix}\boldsymbol{E}_t\\ i\boldsymbol{H}_t\end{bmatrix}=0. \quad (5)$$

Once solutions to (5) are obtained, the corresponding longitudinal field components can be determined from a knowledge of the transverse components via

$$\begin{bmatrix}E_z\\ iH_z\end{bmatrix}=\begin{bmatrix}-\frac{1}{\epsilon_z}\boldsymbol{\varepsilon}_{zt} & \frac{1}{\omega\epsilon_z}\boldsymbol{z}_0\times\boldsymbol{\nabla}_t\\ \frac{1}{\omega\mu_z}\boldsymbol{z}_0\times\boldsymbol{\nabla}_t & -\frac{1}{\mu_z}\boldsymbol{\mu}_{zt}\end{bmatrix}\cdot\begin{bmatrix}\boldsymbol{E}_t\\ i\boldsymbol{H}_t\end{bmatrix} \quad (6)$$

In general, to obtain solutions to the transverse vector eigenvalue problem (5) is a formidable task. We recall that even in the case of isotropic waveguides such solutions are usually obtained by replacing the vector eigenvalue problem by a pair of scalar eigenvalue problems whose eigenfunctions are (except in the case of TEM modes) proportional to the longitudinal field components. A similar technique may be employed in the general anisotropic situation under consideration here. It can be shown that the transverse field components are derivable from the longitudinal field components via

$$D(\kappa)\begin{bmatrix}\boldsymbol{E}_t\\ i\boldsymbol{H}_t\end{bmatrix}=\mathfrak{A}\mathfrak{B}\begin{bmatrix}E_z\\ iH_z\end{bmatrix} \quad (7)$$

where

$$D(\kappa)=\kappa^4+\omega^2\kappa^2\,\mathrm{Tr}\,(\boldsymbol{z}_0\times\boldsymbol{\mu}_t\cdot\boldsymbol{z}_0\times\boldsymbol{\varepsilon}_t)+\omega^4\Delta_\epsilon\Delta_\mu, \quad (8)$$

$$\mathfrak{A}=k^2\Delta_\epsilon\Delta_\mu\begin{bmatrix}\omega\boldsymbol{\varepsilon}_t^{-1} & i\kappa\boldsymbol{\varepsilon}_t^{-1}\cdot\boldsymbol{z}_0\times\boldsymbol{\mu}_t^{-1}\\ i\kappa\boldsymbol{\mu}_t^{-1}\cdot\boldsymbol{z}_0\times\boldsymbol{\varepsilon}_t^{-1} & \omega\boldsymbol{\mu}_t^{-1}\end{bmatrix}+\kappa^2\begin{bmatrix}\omega\boldsymbol{z}_0\times\boldsymbol{\mu}_t\times\boldsymbol{z}_0 & -i\kappa\boldsymbol{z}_0\times\mathbf{1}_t\\ -i\kappa\boldsymbol{z}_0\times\mathbf{1}_t & \omega\boldsymbol{z}_0\times\boldsymbol{\varepsilon}_t\times\boldsymbol{z}_0\end{bmatrix}, \quad (9)$$

$$\mathfrak{B}=\begin{bmatrix}-\omega\boldsymbol{\varepsilon}_{tz} & \boldsymbol{\nabla}_t\times\boldsymbol{z}_0\\ \boldsymbol{\nabla}_t\times\boldsymbol{z}_0 & -\omega\boldsymbol{\mu}_{tz}\end{bmatrix}, \quad (10)$$

Δ_ϵ and Δ_μ are the determinants of (the matrix representations of) the $\boldsymbol{\varepsilon}_t$ and $\boldsymbol{\mu}_t$ dyadics, respectively, and $\mathrm{Tr}\,(\boldsymbol{z}_0\times\boldsymbol{\mu}_t\cdot\boldsymbol{z}_0\times\boldsymbol{\varepsilon}_t)$ is the trace of (the matrix representation for) the dyadic $\boldsymbol{z}_0\times\boldsymbol{\mu}_t\cdot\boldsymbol{z}_0\times\boldsymbol{\varepsilon}_t$. Further, it can be shown that the longitudinal field components satisfy the following pair of (coupled) second-order differential equations (scalar eigenvalue problem):

$$\begin{bmatrix}\epsilon_zE_z\\ i\mu_zH_z\end{bmatrix}=\widehat{\mathfrak{B}}\frac{\mathfrak{A}}{D(\kappa)}\mathfrak{B}\begin{bmatrix}E_z\\ iH_z\end{bmatrix} \quad (11)$$

where $D(\kappa)$, $\mathfrak{A}$, $\mathfrak{B}$ are defined in (7)–(9) and:

$$\widehat{\mathfrak{B}}=\begin{bmatrix}-\omega\boldsymbol{\varepsilon}_{zt} & \boldsymbol{z}_0\times\boldsymbol{\nabla}_t\\ \boldsymbol{z}_0\times\boldsymbol{\nabla}_t & -\omega\boldsymbol{\mu}_{zt}\end{bmatrix}. \quad (12)$$

Note that, in general, $1/D(\kappa)$ does not commute with either $\mathfrak{B}$ or $\widehat{\mathfrak{B}}$ since these contain differentiation operations. The reader may verify that the result in (11) reduces to the equation given by Kales[2] for the special case of an axially magnetized gyromagnetic medium (*i.e.*, where ϵ is a scalar and $\boldsymbol{\mu}_{tz}=\boldsymbol{\mu}_{zt}=0$).

Any solution E_x, H_z to (11) yields, via (7), an eigenfunction (mode) of the transverse vector eigenvalue problem (5). This procedure is manifestly not valid when $D(\kappa)=0$. Therefore, the set of vector eigenfunctions obtained from all the solutions to (11) becomes complete only when we add such vector eigenfunctions of (5) which are admitted when $D(\kappa)=0$. That these additional eigenfunctions are the analogs of the TEM modes in the anisotropic case is evident from the fact that $D(\kappa)=(\omega^2\mu\epsilon-\kappa^2)^2$ for an isotropic medium with scalar μ and ϵ. The analogy to TEM modes indicated here should not be taken to imply any TEM-like properties of these eigenfunctions in the anisotropic case.

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[1] A. D. Bresler, "Vector Formulations for the Electromagnetic Field Equations in Uniform Waveguides Containing Anisotropic Media," Microwave Res. Inst., Polytechnic Inst. of Brooklyn, Brooklyn, N. Y., Rep. R-676-58; September, 1958.

[2] M. L. Kales, "Modes in waveguides that contain ferrites," *J. Appl. Phys.*, vol. 24, pp. 604–608; May, 1953.

An Extension of the Reflection Coefficient Chart to Include Active Networks*

INTRODUCTION

At a single frequency, a two-port can be represented by the scattering matrix [1], [5]

$$[b]=[S][a] \quad (1a)$$

$$b_1=s_{11}a_1+s_{12}a_2 \quad (1b)$$

$$b_1=s_{21}a_1+s_{22}a_2 \quad (1c)$$

where $s_{12}=s_{21}$ in the reciprocal two-port. If one defines an input reflection coefficient $\Gamma_{in}=b_1/a_1$ and a load reflection coefficient $\Gamma_L=a_2/b_2$ one can form

$$\Gamma_{in}=\frac{(s_{12}^2-s_{11}s_{22})\Gamma_L+s_{11}}{1-s_{22}\Gamma_L}. \quad (2)$$

Eq. (2) can be considered as a mapping of the Γ_L plane into the Γ_{in} plane. Since this is a bilinear transformation, angles between

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